

Paper Reference 9FM0/02
Pearson Edexcel
Level 3 GCE

Further Mathematics
Advanced
PAPER 2: Core Pure Mathematics 2

Time: 1 hour 30 minutes

YOU MUST HAVE
Mathematical Formulae and Statistical Tables (Green),
calculator

YOU WILL BE GIVEN
Diagram Booklet
Answer Booklet

Q71801A

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the spaces provided in the Answer Booklet – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

There are 9 questions in this Question Paper. The total mark for this paper is 75

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. Refer to the information for Question 1 in the Diagram Booklet.

A student was asked to answer the following:

For the complex numbers $z_1 = 3 - 3i$ and $z_2 = \sqrt{3} + i$, find the value of $\arg\left(\frac{z_1}{z_2}\right)$

The student's attempt is shown in the Diagram Booklet.

The student made errors in line 1 and line 3

Correct the error that the student made in

(a) (i) line 1

(ii) line 3

(2 marks)

(b) Write down the correct value of $\arg\left(\frac{z_1}{z_2}\right)$
(1 mark)

(Total for Question 1 is 3 marks)

Turn over

2. In this question you must show all stages of your working.

A college offers only three courses: Construction, Design and Hospitality.

Each student enrolls on just one of these courses.

In 2019, there was a total of 1110 students at this college.

There were 370 more students enrolled on Construction than Hospitality.

In 2020 the number of students enrolled on

- Construction INCREASED by 1.25%
- Design INCREASED by 2.5%
- Hospitality DECREASED by 2%

(continued on the next page)

Turn over

2. continued.

In 2020, the total number of students at the college increased by 0.27% to 2 significant figures.

(a) (i) Define, for each course, a variable for the number of students enrolled on that course in 2019

(ii) Using your variables from part (a)(i), write down THREE equations that model this situation.

(4 marks)

(b) By forming and solving a matrix equation, determine how many students were enrolled on each of the three courses in 2019

(4 marks)

(Total for Question 2 is 8 marks)

3. $\mathbf{M} = \begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}$ where a is a constant

(a) Prove by mathematical induction that, for $n \in \mathbb{N}$

$$\mathbf{M}^n = \begin{pmatrix} 3^n & \frac{a}{2}(3^n - 1) \\ 0 & 1 \end{pmatrix}$$

(6 marks)

(continued on the next page)

3. continued.

Triangle **T** has vertices **A**, **B** and **C**

Triangle **T** is transformed to triangle **T'** by the transformation represented by \mathbf{M}^n where $n \in \mathbb{N}$

Given that

- triangle **T** has an area of 5 cm^2
- triangle **T'** has an area of 1215 cm^2
- vertex **A**(2, −2) is transformed to vertex **A'**(123, −2)

(b) determine

(i) the value of n

(ii) the value of a

(5 marks)

(Total for Question 3 is 11 marks)

Turn over

4. (i) Given that

$$z_1 = 6e^{\frac{\pi}{3}i} \text{ and } z_2 = 6\sqrt{3}e^{\frac{5\pi}{6}i}$$

show that

$$z_1 + z_2 = 12e^{\frac{2\pi}{3}i}$$

(3 marks)

(ii) Given that

$$\arg(z - 5) = \frac{2\pi}{3}$$

determine the least value of $|z|$ as z varies.

(3 marks)

(Total for Question 4 is 6 marks)

5. (a) Given that

$$y = \arcsin x \quad -1 \leq x \leq 1$$

show that

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

(3 marks)

(b) $f(x) = \arcsin(e^x) \quad x \leq 0$

Prove that $f(x)$ has no stationary points.

(3 marks)

(Total for Question 5 is 6 marks)

6. The cubic equation

$$4x^3 + px^2 - 14x + q = 0$$

where p and q are real positive constants, has roots α , β and γ

Given that

$$\alpha^2 + \beta^2 + \gamma^2 = 16$$

(a) show that $p = 12$

(3 marks)

Given that

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{14}{3}$$

(b) determine the value of q

(3 marks)

(continued on the next page)

6. continued.

Without solving the cubic equation,

- (c) determine the value of
 $(\alpha - 1)(\beta - 1)(\gamma - 1)$
(4 marks)

(Total for Question 6 is 10 marks)

7. Refer to the diagram for Question 7 in the Diagram Booklet.

It shows a sketch of the curve **C** with equation

$$r = 1 + \tan \theta \qquad 0 \leq \theta < \frac{\pi}{3}$$

The diagram also shows the tangent to **C** at the point **A**

This tangent is perpendicular to the initial line.

- (a) Use differentiation to prove that the polar coordinates of **A** are $\left(2, \frac{\pi}{4}\right)$
(4 marks)

The finite region **R**, shown shaded in the diagram, is bounded by **C**, the tangent at **A** and the initial line.

- (b) Use calculus to show that the exact area of **R** is $\frac{1}{2}(1 - \ln 2)$
(6 marks)

(Total for Question 7 is 10 marks)

8. Two birds are flying towards their nest, which is in a tree.

Relative to a fixed origin, the flight path of each bird is modelled by a straight line.

In the model, the equation for the flight path of the first bird is

$$\underline{r}_1 = \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ a \\ 0 \end{pmatrix}$$

and the equation for the flight path of the second bird is

$$\underline{r}_2 = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

where λ and μ are scalar parameters and a is a constant.

(continued on the next page)

Turn over

8. continued.

In the model, the angle between the birds' flight paths is 120°

**(a) Determine the value of a
(4 marks)**

**(b) Verify that, according to the model, there is a common point on the flight paths of the two birds and find the coordinates of this common point.
(5 marks)**

(continued on the next page)

8. continued.

The position of the nest is modelled as being at this common point.

The tree containing the nest is in a park.

The ground level of the park is modelled by the plane with equation

$$2x - 3y + z = 2$$

(c) Hence determine the shortest distance from the nest to the ground level of the park.

(3 marks)

(d) By considering the model, comment on whether your answer to part (c) is reliable, giving a reason for your answer.

(1 mark)

(Total for Question 8 is 13 marks)

Turn over

9. $y = \cosh^n x \quad n \geq 5$

(a) (i) Show that

$$\frac{d^2 y}{dx^2} = n^2 \cosh^n x - n(n-1) \cosh^{n-2} x$$

(4 marks)

(ii) Determine an expression for

$$\frac{d^4 y}{dx^4}$$

(2 marks)

(b) Hence determine the first three non-zero terms of the Maclaurin series for y , giving each coefficient in simplest form.

(2 marks)

(Total for Question 9 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
